## Lesson 21

If z = f(x, y), then we have differentials  $\partial z$ ,  $\partial x$ , and  $\partial y$ , which are related by the following formula:  $\partial z = f_x(x, y)\partial x + f_y(x, y)\partial y$  (this is called the *total differential*).

We can approximate  $\Delta z$  by  $\Delta z \approx f_x(x, y)\Delta x + f_y(x, y)\Delta y$ .



1. Find  $\partial z$  for  $z = e^{x^2 + y^2} \tan(2x)$ .

- 2. Use the total differential to approximate the quantity  $\sqrt[3]{2.6^2 1.07^2} \sqrt[3]{2.5^2 1^2}$  to 3 decimal places.
  - (a) Fill in the table:

f(x,y)	
$f_x$	
$f_y$	
x	
y	
$\Delta x$	
$\Delta y$	

(b) Approximate the quantity  $\sqrt[3]{2.6^2 - 1.07^2} - \sqrt[3]{2.5^2 - 1^2}$ .

3. A company's profit is given by  $P(K, L) = 500K^{1/3}L^{2/3}$ , where K is the company's overhead costs in thousands and L is the number of workers in hundreds. Find the change in profit when the overhead costs are currently 3 million dollars and there are 2,500 workers and overhead costs are decreased by 5 thousand dollars while the number of workers is increased by 150.

P(K,L)	
$P_K = \frac{\partial P}{\partial K}$	
$P_L = \frac{\partial P}{\partial L}$	
K	
L	
$\Delta K$	
$\Delta L$	

(a) Fill in the table:

(b) Approximate  $\Delta P$ .

- 4. Recall that  $A = Pe^{rt}$ . Suppose you deposit \$1,000 today into a 5 year CD and interest is compounded continuously at an annual rate of 2%.
  - (a) How much money will be in the CD after 5 years?

	¥	¥
A(P,r)		
$A_P = \frac{\partial A}{\partial P}$		
$A_r = \frac{\partial A}{\partial r}$		
Р		
r		
$\Delta P$		
$\Delta r$		

(b) Suppose the rate changes to 1.95%. Fill in the following table:

(c) Approximately how much more will you need to deposit today to obtain the same amount in 5 years?

- 5. A tank is a cylinder h feet tall with radius r feet. Recall the surface area of a cylinder with no top is  $A(r, h) = \pi r 2 + 2\pi r h$ . A particular tank is measured to be 6 feet tall with a radius of 3 feet. The height is measured with an error of at most 3 inches (1/4 of a foot) and the radius is measured with a maximum error of 1 inch (1/12 of a foot).
  - A(h,r) 

      $A_h = \frac{\partial A}{\partial h}$ 
     $A_r = \frac{\partial A}{\partial r}$  

     h 

     r 

      $\Delta h$ 
     $\Delta r$
  - (a) Fill in the following table:

(b) What is the maximum error in the calculation of the surface area?

(c) What is the relative percentage error in calculating A?